Simulation-based quantitative methods for vehicle emissions and a CO_2 charging policy

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ABSTRACT

With the worsening vehicle emissions, carbon emission charges are becoming an increasingly popular policy to reduce emissions. This paper proposes a policy to charge for the additional CO_2 emissions due to the increasing traffic flow for each vehicle, thereby extending traditional travel time-based traffic assignment to emissions-charged traffic assignment. Considering that vehicle movements in the network are affected by signal timings and the car-following together with lane-changing interactions of different types of vehicles, microscopic traffic flow simulation is combined with a CO_2 emission model to formulate the relations between link flow and emissions of different types of vehicles. Accordingly, the additional CO_2 emissions due to increasing traffic flow are quantified and charged for each vehicle, leading to multi-vehicle-type and multi-criteria traffic assignment. Through an example network, the proposed flow-emissions model is verified and the impacts of different CO_2 charging prices in the morning peak hour are investigated. Analysis of the results shows monotone increasing relations between price and reduced total emissions as well as increased revenue within the price range. In addition, the charging policy also leads to traffic assignment that achieves a system-optimal assignment since the CO_2 pricing aligns with the general concept of pricing externality, thereby reducing the total travel time.

KEYWORDS Stochastic user equilibrium; traffic flow simulation; CO_2 emission model; flow-emissions relations; additional emissions; CO_2 charging policy
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Received 29 November 2022

1. Introduction

CO_2 emissions are major sources of greenhouse gases. Of worldwide CO_2 emissions 21% are caused by the transport sector, of which road vehicles account for 74.5%. Reducing CO_2 emissions has been an important aim of traffic assignment optimisation as global warming has become an increasingly serious problem. The development practices of many cities show that a CO_2 charging policy can effectively encourage vehicle drivers to choose low-emissions routes on the basis of meeting the original traffic demand, thus reducing the total CO_2 emissions in the network.

Routing traffic demand under certain behavioural principles in a network is called traffic assignment. Traditional Wardrop's first principle and second principle (Wardrop, 1952) are used to achieve user equilibrium (UE) and a system-optimal (SO) assignment respectively. UE specifies that all users choose routes to minimise their own travel costs while a SO assignment assumes that all users minimise the total travel cost. Accordingly, these two principles have different mathematical formulations. Dafermos (1969) developed a method of adding the marginal cost of travel on each link to achieve a SO assignment by calculating UE with modified link costs, which is also considered as internalising the externality. As a pure equilibrium, UE assumes that users can accurately calculate the actual cost of each route and then choose the least costly route accordingly. However, in practice, there is a perception error between the perceived time and the actual time and hence stochastic user equilibrium (SUE) is achieved, which still follows Wardrop's first principle but is based on perceived route costs (Daganzo, 1977). In the research on the SUE problem, the Multinomial Logit (MNL) model (McFadden, 1974) is widely used but it has some disadvantages such as the independence from irrelevant alternatives (IIA) property (Train, 2003), which cannot reflect the impacts of path overlap on user selection. And many studies have not considered the situation of multiple vehicle types under multiple criteria (time cost and monetary expenditure), resulting in a large discrepancy between the calculated assignment results and the actual assignment.

Like travel time, vehicle emissions can also conveniently be modelled as increasing with the flow and depending only on the flow on a link. Patil (2016) deduced a macroscopic flow-average emissions relationship from a traditional flow-average speed model together with fitted emissions-average speed relationships. However, some studies have proved that not only vehicle speed, acceleration, deceleration, and corresponding vehicle-following interactions also play major roles in the amount of emissions by a vehicle (Jackson and Aultman-Hall, 2010). Meanwhile, the differences of vehicle power sources and vehicle types also have certain impacts on carbon emissions. Considering an aggregated behaviour for different groups of vehicles, Jamshidnejad et al. (2017) later proposed a mesoscopic flow-emissions model by interfacing and integrating macroscopic flow models and microscopic emission models. Furthermore, in order to explore the impacts of signal timing on vehicle emissions, Spiropoulou (2007) simulated vehicle trajectories according to Gipps' car-following model (Gipps, 1981) in a signal-controlled traffic stream, in which the motion states of each vehicle at every time step have different emission rates. Li et al. (2011) also investigated the impacts of signal timing on vehicle emissions at an isolated junction. Han et al. (2016) proposed approaches of traffic signal control to reduce emissions. These models explore the impacts of different factors on vehicle emissions while they lack research on the combined effects of these factors, especially the multiple types of vehicles on an actual multiple-lane link connecting two signal-controlled junctions.

Environmental factors such as CO_2 emissions can be incorporated as the objectives or constraints in traffic assignment models to meet the requirements of the environment, but they all need the assistance of traffic control to achieve the ideal assignment. However, some relevant studies show that adding CO_2 charges to travel costs enable vehicle drivers to spontaneously choose routes on which vehicles emit less CO_2, thereby reducing the total CO_2 emissions in the network (Yin and Lawphongpanich, 2006). In practice, the governments and transportation management departments in some countries and regions have implemented CO_2 taxes and charging policies on road systems to reduce carbon emissions in transportation. A few European countries have levied taxes on vehicles based on the amount of CO_2 emissions, but the tax rates vary greatly among different countries. For example, such taxes were set at rates of US$20.3 and US$137.2 (in equivalent currency) per tonne of CO_2 in Slovenia and Sweden respectively in 2021, and the rate is generally above US$50 in developed countries (The World Bank, 2021). In addition, in February 2022, the European Parliament approved distance-based road charging rules for heavy goods vehicles to reduce CO_2 emissions. Member states can impose charges on trucks and lorries according to their actual kilometres driven. The rules also require EU countries to set different road charging rates based on CO_2 emissions and environmental performance for different vehicles from 2026 (European Parliament, 2022). Although CO_2 amount-based charging policies are relatively fair, it is difficult and costly to directly measure vehicle emissions. Only a few countries such as Poland and the Czech Republic adopt this approach while more countries still use theoretical calculation methods. Furthermore, a new charging policy for all CO_2 emissions is also unacceptable for a region that has not implemented any CO_2 charges. Therefore, economic methods for quantifying vehicle CO_2 emissions and a fair, progressive as well as acceptable charging policy instead of copying the stereotyped policies need to be proposed.

In view of these above-mentioned research gaps, under a more realistic multi-vehicle-type traffic assignment, it is necessary to economically formulate accurate relationships between link flow and the emissions of different types of vehicles considering the impacts of signal timings, vehicle motion parameters of different vehicles and car-following together with lane-changing interactions on emissions, which is aimed at proposing a charging policy for each link to only charge for the additional emissions due to increasing traffic flow. At the same time, if the charging policy aligns with the concept of pricing externality, it can also reduce the total travel time to some extent.

Accordingly, this paper proposes simulation-based quantitative methods for vehicle emissions and a CO_2 charging policy. For each link in the network, the additional emissions due to increasing traffic flow will be charged as CO_2 cost and added to each link cost function, which leads vehicles to choose lower-emissions routes on the basis of meeting the original traffic demand. Firstly, the relations between link flow and the average vehicle CO_2 emissions of different types of vehicles need to be investigated. According to a peak-hour multi-vehicle-type and multi-criteria (time cost and monetary expenditure) stochastic assignment with link flows, the network-wide traffic signal timings are set at each node, with simulation of the flow in each link between two junctions by microscopic vehicle-following combined with a lane-changing model. Accordingly, the carbon emission model respectively aggregates the CO_2 emissions from each vehicle over the link and the average vehicle CO_2 emissions of different types of vehicles under the link flow are calculated. After that, by adjusting the network demand data and repeating the above steps, for each link, the data sets of link flows and the average vehicle CO_2 emissions of different types of vehicles are collected in order to obtain the regression model that can formulate the CO_2 emissions under a link flow. Accordingly, the additional CO_2 emissions that increase with link flow are charged a reasonable price and added to the link cost function with different time values in terms of the vehicle type, leading to an optimised lower-emissions traffic assignment. Finally, an objective comparison between the present model and a measurement-based model is made and the performances of different CO_2 pricing levels in the Sioux Falls network are illustrated.

It should be noted that although the stochastic assignment model applied in this paper is both static and macroscopic, its function is to obtain the network-wide link flow assignment and signal timings in a peak hour. After that, based on the assumption that the traffic flow is uniformly distributed during this hour and the above-mentioned simulation methods, for each link, the data sets of link flows and corresponding average vehicle emissions of different types of vehicles are collected in order to obtain their regression models, which is the key to reflect the accurate and realistic relationships between them in a real traffic network.

2. Methodology

In this section, simulation-based quantitative methods are devised to formulate the relations between link flow and the average CO_2 emissions from different types of vehicles for each link in a network and a CO_2 charging policy considering the time value to charge for the additional emissions is proposed.

2.1. Multi-vehicle-type and multi-criteria stochastic user equilibrium

McFadden (1978) proposed the Generalised Extreme Value (GEV) theory, which provides a broader space for the development of path selection models, and many new path selection models have been proposed and applied successively. As one of the GEV models, the Generalised Nested Logit (GNL) model (Wen and Koppelman, 2001) has the characteristics of capturing similarities among schemes and can overcome the IIA property of MNL models. This paper applies a two-level nested structure to explore this multi-vehicle-type and multi-criteria GNL model for the time costs and monetary expenditures of different vehicle types with different time values on a trip. The upper level is all the links (nests) in the traffic network and the lower level is all the efficient paths between OD pairs. Each path can be assigned to different nests and the non-positive allocation parameter α_{mk}^{rs} characterises the portion of an alternative path assigned to a certain nest. The probability of selecting path k between OD pair rs for user class i is as follows.

$$P_k^{i,rs} = \frac{\sum_m \left[\left(\alpha_{mk}^{rs} \exp\left(-\theta c_k^{i,rs}\right)\right)^{1/\mu_m^{rs}} \left(\sum_k \left(\alpha_{mk}^{rs} \exp\left(-\theta c_k^{i,rs}\right)\right)^{1/\mu_m^{rs}}\right)^{\mu_m^{rs}-1}\right]}{\sum_m \left(\sum_k \left(\alpha_{mk}^{rs} \exp\left(-\theta c_k^{i,rs}\right)\right)^{1/\mu_m^{rs}}\right)^{\mu_m^{rs}}}, \quad (1)$$

$$\alpha_{mk}^{rs} = (L_m / L_k^{rs})^{\tau} \delta_{mk}^{rs}, \quad (2)$$

$$\mu_m^{rs} = 1 - \frac{1}{T_m} \sum_k \alpha_{mk}^{rs}, \quad (3)$$

where $c_k^{i,rs}$ is the generalised cost of path k for vehicle type i between the od pair rs, θ is a positive dispersion parameter reflecting the user's familiarity with the traffic network, α_{mk}^{rs} represents the ratio of path k assigned to nest m, L_m is the length of link m, L_k^{rs} is the length of path k, τ is a parameter reflecting the user's perception of similarities among paths, $\delta_{mk}^{rs} = 1$ if link m belongs to path k (0, otherwise), μ_m^{rs} is a parameter indicating the similarities among nests, and T_m is the number of paths between od pair rs containing link m.

According to Equation (1), the conditional probability of alternative path k if nest m is selected is derived as follows.

$$P^{i,rs}(k|m) = \frac{\left(\alpha_{mk}^{rs} \exp\left(-\theta c_k^{i,rs}\right)\right)^{1/\mu_m^{rs}}}{\sum_k \left(\alpha_{mk}^{rs} \exp\left(-\theta c_k^{i,rs}\right)\right)^{1/\mu_m^{rs}}}. \quad (4)$$

In this paper, the generalised travel cost is measured by time. Accordingly, the cost of link a for vehicle type i is given by Equation (5), which contains the widely-used US Bureau of Public Road (BPR) function (1964) and time cost converted from the monetary expenditure, and the cost of path k between rs for vehicle type i is calculated as Equation (6).

$$t_a^i = t_a^0 \left[1 + \alpha (v_a / Q_a)^{\beta}\right] + \phi^i p_a^i \quad \forall a \in A, \quad (5)$$

$$c_k^{i,rs} = \sum_a \left[t_a^0 \left(1 + \alpha (v_a / Q_a)^{\beta}\right) + \phi^i p_a^i\right] \delta_{a,k}^{rs} \quad \forall a \in A, \quad (6)$$

where A is the set of links in the network, t_a^0 is the free-flow travel time in link a, v_a is the passenger-car-unit (PCU) traffic flow carried by link a, Q_a is the capacity of link a, ϕ^i is the money-time conversion parameter for vehicle type i, p_a^i is the monetary cost of link a for vehicle type i, $\delta_{a,k}^{rs} = 1$ if link a belongs to path k (0, otherwise), and α, β are parameters which determine the travel-time relationship.

Since the link cost is dependent on link flow, the method of successive averages (MSA) is introduced to achieve the iteration process for solving the stochastic user equilibrium and the steps are designed as follows.

Step 0: Initialisation. Perform a stochastic assignment by Equation (1) based on free-flow $\left\{c_k^{i,rs}\right\}$ calculated from $\left\{t_a^i(0)\right\}$ under od demand $\{T_{od}\}$, obtain the initial path flows $\left\{f_k^{i,rs(0)} = q_{rs}^i P_k^{i,rs(0)}\right\}$ and $\left\{v_a^{(0)} = \sum_{rs} \sum_i \sum_k f_k^{i,rs(0)} \delta_{a,k}^{rs}\right\}$, and set counter $n = 1$;

Step 1: Update. Calculate the link costs $\left\{t_a^{i(n)} = t_a^0 \left[1 + \alpha \left(v_a^{(n)} / Q_a\right)^{\beta}\right] + \phi^i p_a^i\right\}$;

Step 2: Network loading. Perform a stochastic assignment by Equation (1) based on the path costs calculated by $\left\{t_a^{i(n)}\right\}$ from step 1, then obtain the auxiliary path flows $\left\{{f'}_k^{i,rs(n)}\right\}$;

Step 3: Move. Calculate the new path flows $\left\{f_k^{i,rs(n)} = f_k^{i,rs(n-1)} + \frac{1}{n}\left({f'}_k^{i,rs(n)} - f_k^{i,rs(n-1)}\right)\right\}$ and $\left\{v_a^{(n)} = \sum_{rs} \sum_i \sum_k f_k^{i,rs(n)} \delta_{a,k}^{rs}\right\}$;

Step 4: Convergence test. If $\frac{\sqrt{\sum_k \left(f_k^{i,rs(n)} - f_k^{i,rs(n-1)}\right)^2}}{\sum_k f_k^{i,rs(n-1)}} < \varepsilon$ (predetermined tolerance), stop calculating and output $\left\{v_a^{(n)}\right\}$. Otherwise, set $n = n+1$ and return to step 1.

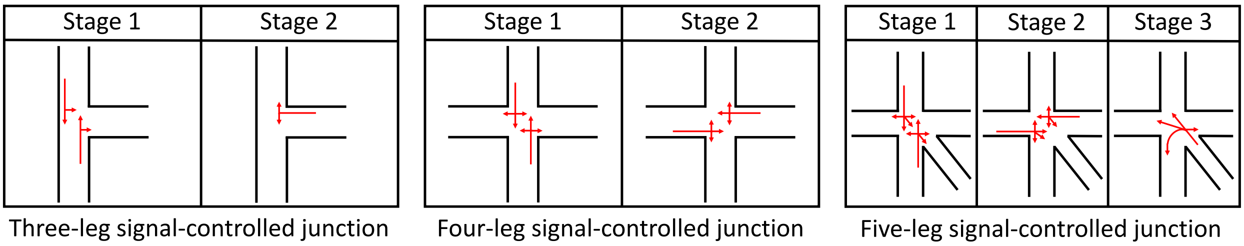


Figure 1. Stages and phases of different signal-controlled junctions.

2.2. Traffic flow simulation between two junctions

The above-designed algorithms provide an efficient method to compute a static multi-vehicle-type and multi-criteria stochastic assignment with link flows. In this part, based on the consideration of traffic signals at both ends of the link, an approach is provided for loading the traffic flows on the link and simulating vehicles' microscopic motions.

2.2.1. Traffic signal timing and signal coordination

In a real traffic network, most junctions are signal-controlled and few are flyover-designed without signals. For signal-controlled junctions, there are several common types consisting of three-leg junctions, four-leg junctions and five-leg junctions. In terms of calculating the cycle time, identifying the stages and the phases included in every stage of each type of signal-controlled junctions as shown in Figure 1 is essential.

Accordingly, the cycle times and green splits of common signal-controlled junctions can be calculated according to Webster's formulae (1958) as follows.

$$c_0 = \frac{1.5L^c+5}{1-Y}\,, \tag{7}$$

$$Y = \sum_{j=1}^{K} y_j\,, \tag{8}$$

$$y_j = \max\left\{\frac{q_i}{s_i}\right\}, \tag{9}$$

$$g_j = (c_0 - L^c)\frac{y_j}{Y}\,, \tag{10}$$

where L^C is the total lost time in a cycle (generally 3 sec/stage), Y is the summation over different stages, y_j is the biggest ratio of the demand and saturation flow of phases in stage j, q_i is the demand of phase i, s_i is the saturation flow of phase i, and g_j is the corresponding green duration in stage j.

In addition, signal coordination is widely applied for closely spaced junctions. For two opposite links between two adjacent junctions, as shown in Figure 2, with SUE link flows (v_a^{SUE} and v_b^{SUE}) and corresponding link travel times (t_a^{SUE} and t_b^{SUE}), the green offset Δg is determined by the busier link that has a higher ratio of the flow and the capacity. According to the two calculated cycle times of the adjacent junctions, for the busier link, the offset setting should satisfy that the first entering vehicle from the upstream junction during the green signal will meet the start of the green signal of the downstream junction, as shown in Equation (11).

$$\Delta g = \mathrm{MOD}(t_m^{SUE}, c_d)\,, \tag{11}$$

where t_m^{SUE} is the SUE travel time on the busier link and c_d is the signal cycle time of the downstream junction of the busier link.

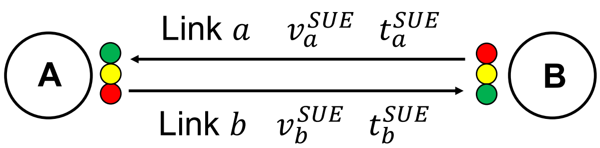


Figure 2. Opposite links between two junctions.

2.2.2. Microscopic simulation between two junctions

After setting the traffic signal timings in the whole network, Gipps' car-following model can simulate the loaded flow on a multiple-lane link under the assumption that vehicles do not change their driving lane. This model is a behavioural car-following model that is discrete in time but continuous in space. The speed at each time step is determined by the acceleration (or when negative, braking) at the previous time step. In terms of the parameters in this model, for vehicle n, they consist of minimum spacing s_n, acceleration a_n, maximum braking b_n, and desired speed $\bar{v}_n$. Here, s_n ensures that a vehicle can stop a safe distance away from the front vehicle. There are also braking $\hat{b}$ of the preceding vehicle (estimated by the driver of vehicle n) and Δt representing the reaction time of drivers. Meanwhile, different types of vehicles have a corresponding set of parameters. According to these features, the equations for Gipps' model are as follows.

$$a_n^+ = \alpha_G a_n \sqrt{\beta_G + \frac{v_n(t)}{\bar{v}_n}} \left(1 - \frac{v_n(t)}{\bar{v}_n}\right), \quad (12)$$

$$a_n^- = b_n + \sqrt{b_n^2 - b_n\left[\frac{2(x_{n-1}(t) - s_{n-1} - x_n(t))}{\Delta t^2} - \frac{v_n(t)}{\Delta t} - \frac{(v_{n-1}(t))^2}{\hat{b}\Delta t^2}\right]}, \quad (13)$$

$$v_n(t+\Delta t) = v_n(t) + \mathrm{Min}(a_n^+, a_n^-)\Delta t, \quad (14)$$

$$x_n(t+\Delta t) = x_n(t) + \frac{1}{2}[v_n(t) + v_n(t+\Delta t)]\Delta t, \quad (15)$$

where Equation (12) and Equation (13) represent the acceleration part and deceleration part respectively. The minimum value of them is used for calculating the updated speed, indicating that drivers are represented as wishing to travel as quickly as possible subject to safety. In addition, the relationship between these two parts also indicates the traffic state: when $a_n^+ < a_n^-$, the vehicle is free; if $a_n^+ > a_n^-$, the vehicle is hindered.

In a real traffic network, a link between two adjacent junctions always contains several lanes. Considering the lane changing behaviours of vehicles, this paper combines the Gipps car-following model with the lane-changing model developed by Gu et al. (2020). Based on the driver's sensitivity and pursuit of the "ideal speed", the lane-changing behaviours are possible with a certain probability π when the necessity condition and safety condition are met respectively as follows.

$$a_n > a_{n-1} \text{ and } G_f < G_{cd}^i, \quad (16)$$

$$G_f{}' > G_{sf}^i{}' \text{ and } G_p{}' > G_{sp}^i{}', \quad (17)$$

where a_n is the acceleration of vehicle n, a_{n-1} is the acceleration of the vehicle in front of vehicle n, G_f is the distance from the front vehicle, G_{cd} is the acceptable distance of vehicle type i from the front vehicle, $G_f{}'$ and $G_p{}'$ represent the distance from the front/back vehicle respectively at the lane-changing position in the adjacent lane, and $G_{sf}^i{}'$ and $G_{sp}^i{}'$ is the safe distance of vehicle type i from the front/back vehicle respectively in the lane-changing position.

According to the above-designed methods, for a link connecting two signal-controlled junctions with calculated corresponding cycle times (c_A and c_B), green splits (g_{Aj} and g_{Bj}) toward the link, and a green offset Δg, the simulation scheme between two junctions based on Gipps' model combined with a lane-changing model is shown in Figure 3. This simulation assumes: (i) the waiting vehicles from red indication time will dissipate clearly during the green indication time, (ii) the traffic flow is uniformly distributed during the simulation period, (iii) the proportions of different types of vehicles are certain and fixed, (iv) different types of vehicles are randomly distributed in the traffic flow, (v) the possibility of lane changing for each vehicle is equal when the lane-changing conditions are met during the simulation, and (vi) junctions which are not signal-controlled are considered as constantly green and the speeds of vehicles crossing them will not be affected.

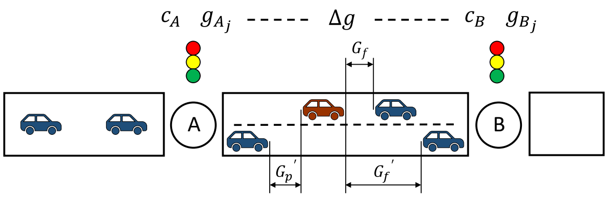


Figure 3. Simulation scheme between two junctions.

The simulation duration t_{in} and the initial number N_{in}^l of vehicles entering each lane during the period are calculated as follows.

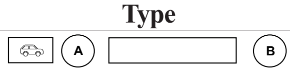
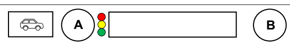
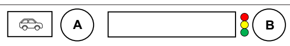
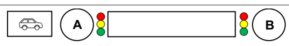
$$t_{in} = c_{AB} + \Delta g, \quad (18)$$

$$N_{in}^l = \sum_i \frac{q_{in}^i}{\vartheta_i} t_{in} \frac{1}{N_l}, \quad (19)$$

where c_{AB} is the least common multiple cycle of c_A and c_B, q_{in}^i is the inflow of vehicle type i toward the simulated link (equal to the PCU flow of vehicle type i in the SUE link flow v^{SUE}), N_l is the number of lanes in the link, and ϑ_i is the PCU conversion parameter for vehicle type i.

In addition, considering the setting of signal lights at both ends of the simulated link, there are four simulation scenarios shown in Table 1. As a pre-requisite condition, it is essential to set the initial states before simulation. For

Table 1. Initial state settings for the four scenarios.

Type	t_{in}	Initial state
	600 sec	Calculated speed by travel time from the BPR function
	C_A	0 speed
	C_B	Desired speed $\bar{v}_n$ from Gipps' car-following model
	$C_{AB} + \Delta g$	0 speed

a link connecting two junctions, before the simulation starts, it is assumed that these vehicles N_{in}^{l} in which the vehicle types are randomly distributed are behind the upstream junction at a certain spacing per vehicle. Once the simulation starts, all vehicles will run according to the above-mentioned car-following and lane-changing model until they finish the simulated link.

2.3. CO_2 emission model

According to the above-designed simulation approaches, for an assigned traffic flow in a link with known attributes, the behaviour values of each vehicle at every time step during simulation are output. These data are input into a CO_2 emission model to compute the CO_2 emissions of each vehicle.

Oguchi et al. (2002) developed an efficient CO_2 emission model that indicates a relationship between fuel consumption and CO_2 emissions. The fuel consumption is divided into three parts in detail for a certain vehicle n: duration part $F_{n,t}$, distance part $F_{n,l}$ and speed variation part $F_{n,a}$. Finally, a coefficient K_c that represents the conversion relationship between fuel consumption and CO_2 emissions according to the vehicle type of vehicle n is introduced to calculate the CO_2 emissions. These parameters can be derived from the experiments in the study area to achieve a high accuracy. As shown in Equations (20)-(23), this approach can be interfaced conveniently with a microscopic simulation of vehicle movements output by the Gipps' car-following model combined with the lane-changing model to compute vehicle emissions.

$$E_n = K_c F_n = K_c\left(F_{n,t} + F_{n,l} + F_{n,a}\right), \tag{20}$$

$$F_{n,t} = f_{i,t} t_n, \tag{21}$$

$$F_{n,l} = f_{i,l} l_n, \tag{22}$$

$$F_{n,a} = f_{i,a}\left(\sum_{k=1}^{K} \delta_k \left({v_{n,k}}^2 - {v_{n,k-1}}^2\right)\right), \tag{23}$$

where for vehicle n (corresponding type i), K_c is the consumption-emissions parameter, $f_{i,t}$ is the consumption-duration parameter, t_n is the interval of duration, $f_{i,l}$ is the consumption-distance parameter, l_n is the driving distance, $f_{i,a}$ is the fuel consumption parameter if there are speed variations, k is the time step k, v_k is the speed at time step k, and $\delta_k = \begin{cases} 1 & v_{n,k} > v_{n,k-1} \\ 0 & v_{n,k} \le v_{n,k-1} \end{cases}$.

2.4. Relation between link flow a nd CO_2 emissions

This study assumes that a reasonable and acceptable CO_2 charging price will not affect the fixed traffic demand, and accordingly, the charging policy will not have a great impact on the traffic assignment. Therefore, the range of data sets for regression do not need to be too large. In this part, four more multi-vehicle-type and multi-criteria stochastic assignments are performed after increasing the network *od* demand by 20% and 10% and decreasing it by 10% and 20%. Accordingly, for each link in the traffic network, the corresponding average vehicle CO_2 emissions of different types of vehicles over the link are calculated according to the above-designed methods under five different link flows. Considering that the emissions are always positive, a log-linear regression model is used to fit the relation between link flow and the average vehicle CO_2 emissions of different types of vehicles. Hence, for each link in the network, its regression model and corresponding transformed form for vehicle type i are shown as Equation (24) and Equation (25) respectively.

$$\mathrm{Log}_e\, {E_a^i}^* = \gamma_a^i + \eta_a^i v_a \quad \forall a \in A, \tag{24}$$

$${E_a^i}^* = \exp\left(\gamma_a^i + \eta_a^i v_a\right) \quad \forall a \in A, \tag{25}$$

where ${E_a^i}^*$ represents the average CO_2 emissions over link a from a vehicle belonging to type i and v_a is the PCU traffic flow carried by link a.

2.5. CO_2-charged link cost function

In terms of charging for CO_2 emissions for different types of vehicles in a link, the necessary CO_2 emissions from a vehicle over the link under free flow (v=0) are excluded, and only the additional CO_2 emissions due to an increased flow in this link will be subject to a charge. However, unlike the BPR function using time units, the price p_{CO2} for charging for CO_2 emissions is in money units. Hence, there is a conversion parameter ϕ^i introduced to convert money cost into time cost in regard to the BPR function according to the different time values of different vehicle types, thereby updating the multi-vehicle-type and multi-criteria CO_2-charged link cost function in Equation (26).

$${t_a^i}^* = t_a^0\left\{1 + \alpha\left(v_a/Q_a\right)^{\beta}\right\} + \phi^i p_{CO2}\left\{\exp\left(\gamma_a^i + \eta_a^i v_a\right) - \exp\left(\gamma_a^i\right)\right\} \quad \forall a \in A. \tag{26}$$

3. Case study

In order to illustrate the performance of the proposed CO_2 charging policy in this paper, the accuracy of the proposed model is verified and applied to an example network to explore the impacts of different CO_2 charging prices on the network in the morning traffic peak.

3.1. The Sioux Falls Network

The Sioux Falls network is shown in Figure 4; this network has 24 nodes, 76 links, 528 *od* pairs, and 716 paths (Leblanc, 1975). In practice, eight flyover-designed nodes including 1, 2, 3, 7, 12, 13, 18 and 20 are not signal-controlled and considered as constantly green.

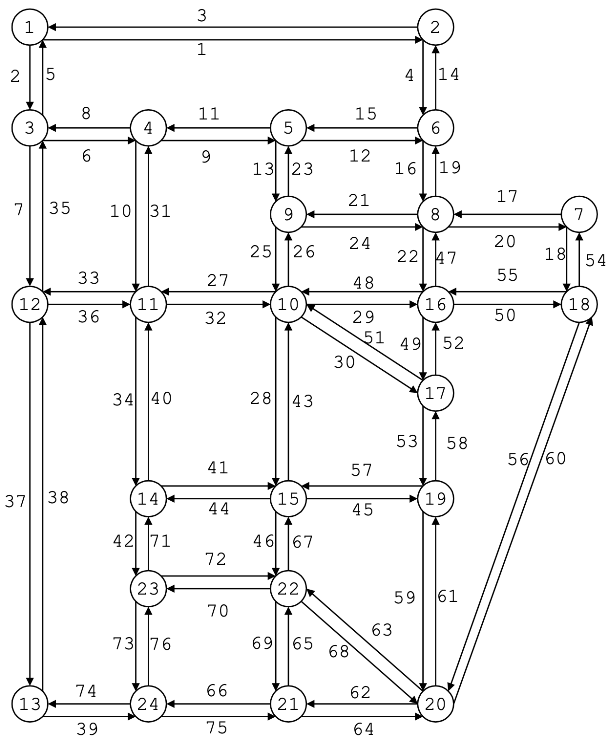


Figure 4. Sioux Falls network.

3.2. Data preparation and parameter setting

This paper explores the effects of the CO_2 pricing policy on traffic between 7.00 am and 8.00 am, which accounts for 24% of daily *od* demand. Furthermore, before implementing the methods designed in section 2, the parameter settings in Table 2 are also required. For different types of vehicles, i = 1, 2, 3, 4, 5 represent small gasoline vehicles, small electric vehicles, small hybrid-electric vehicles, mid-sized vehicles and full-sized vehicles respectively. The corresponding proportions of them are set to 50%, 10%, 15%, 20%, and 5% according to the actual distribution in the study area.

3.3. Results

3.3.1. Relation between link flow and average CO_2 emissions from vehicles

By implementing the methods designed in section 2, the results show that the coefficients of determination of these 380 (five types of vehicles in the 76 links) regression models are all greater than 0.9, and their constants and coefficients are all statistically significant at 90% confidence levels. This indicates that these regression models can compute the simulation-based quantitative results well for the CO_2 emissions of different types of vehicles within the range for each link.

Table 2. Parameter settings.

Sub-model	**Parameter**	**Definition**	**Value**	**Unit**
SUE model	θ	Dispersion parameter	2.0	1/min
	τ	Parameter	1.0	1
MSA method	ε	Predetermined tolerance	0.0001	1
BPR function	α	Determined parameter	0.15	1
	β	Determined parameter	4.0	1
Gipps' model	Δt	Reaction time	2/3	s
	α_G	Constant	2.5	1
	β_G	Constant	0.025	1
	α_i	Maximum acceleration	1.7/2.2/1.9/1.2/0.9	m/s^2
	$\bar{v}_n$	Desired speed	20.0	m/s
	b_i	Maximum braking	-8.0/-8.0/-8.0/-6.5/-4.5	m/s^2
	s_i	Minimum spacing	7.0/7.0/7.0/10.0/18.0	m
Lane-changing model	π	Probability parameter	0.15	1
	G^i_{cd}	Acceptable gap	20.0/20.0/20.0/30.0/35.0	m
	$G^{i\prime}_{sf}$	Safe front-vehicle distance	27.5/27.5/27.5/35.5/45.0	m
	$G^{i\prime}_{sp}$	Safe back-vehicle distance	15.0/15.0/15.0/20.0/25.0	m
PCU conversion	ϑ_i	Conversion parameter	1.0/1.0/1.0/2.0/3.0	1
CO_2 emission model	K_c	Conversion parameter	2.35	kg/L
	$f_{i,t}$	Consumption parameter	0.30/0.00/0.15/0.60/0.90	ml/s
	$f_{i,l}$	Consumption parameter	0.028/0.000/0.015/0.057/0.085	ml/m
	$f_{i,a}$	Consumption parameter	0.056/0.000/0.028/0.113/0.170	$ml{\cdot}s^2/m^2$
CO_2 pricing	ϕ_i	Conversion parameter	1.70/1.50/1.65/1.25/1.75	min/US$

Taking the small gasoline vehicles in link 12 from signal-controlled node 5 to signal-controlled node 6 as an example, the relation between link flow and average CO_2 emissions from vehicles is formulated in Equation (27).

$$E_{12}^{1\ *} = \exp\left(-0.8036 + 8.1349 \times 10^{-6} v_{12}\right). \tag{27}$$

This regression model fits the flow-emissions relationship in link 12 between 80% and 120% of demand. As shown in Figure 5, within the demand range, in terms of small gasoline vehicles, the model proposed in this paper and the model proposed by Patil (2016) based on the actual measurements have a high degree of fit, with an error of less than 3%. After further verification, for other links, the error is within 6%. The reason why the estimated values are generally smaller is that the design of vehicle motions in the simulation is ideal, while in reality there will be more complex vehicle motion behaviours, resulting in an increase in emissions.

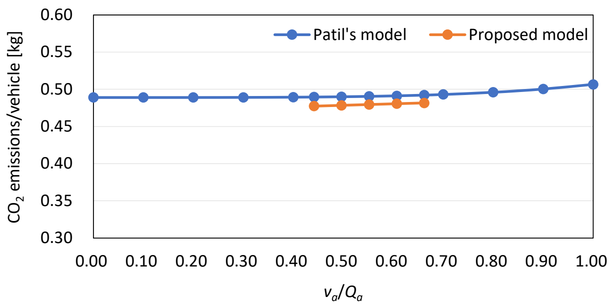


Figure 5. Flow–emissions relationships of Patil's model and the proposed model.

3.3.2. Impacts of different pricing levels on the traffic network

Identifying a reasonable price for charging for CO_2 emissions is a prerequisite for adding CO_2 cost to the link cost. Referring to relevant policies in some countries and regions, High-Level Commission Carbon Prices estimates that only when the CO_2 tax is at least US$40 to 80 per tonne of CO_2, can the goal of emission reduction in the Paris Agreement be achieved in a cost-effective way (The World Bank, 2021). Accordingly, this paper sets the price to US$0.06/kg (US$60 per tonne equivalent) for the example network that has not implemented any CO_2 charges and investigates the impacts of different pricing levels (US$0.02/kg, US$0.04/kg, US$0.06/kg, US$0.08/kg and US$0.10/kg) on the traffic network. The corresponding results are shown in Figure 6.

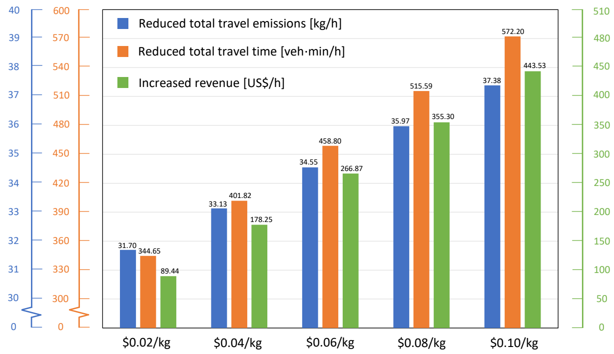


Figure 6. Impacts of different CO_2 pricing levels on the network.

3.4. Result analysis

Based on the assumption of fixed traffic demand, for the peak hour from 7.00 am to 8.00 am, within the set pricing range, as the CO_2 charging price rises, the reduced emissions and reduced total travel time both increase, and the revenue received by the traffic management department also increases.

It is easily accepted that a CO_2 charging policy can lead vehicle drivers to choose routes on which they emit less CO_2, thereby reducing total emissions and achieving revenue for the traffic management departments in a network. The reason for the reduction in total travel time is that the CO_2 pricing aligns with the more general concept of pricing externality. Stochastic user equilibrium is a special case of user equilibrium. Pricing externality will lead to a system-optimal assignment, and the total travel time will continue to decrease until the pricing is optimal.

4. Conclusion and future work

In this paper, a set of simulation-based methods are proposed to formulate a CO_2 charging policy based on the actual conditions of the traffic network in order to charge for additional emissions due to increasing traffic flow. For different types of vehicles, considering the vehicle motion states affected by signal timings, vehicle motion parameters and car-following together with lane-changing interactions, the realistic relations between link flows and average vehicle emissions of different types of vehicles are established for each link. Accordingly, compared with the minimum (free-flow) emissions in each link, the additional emissions due to increasing traffic flow are charged a price based on the link cost function as a charging policy. Furthermore, the flow-emissions model is verified by comparing it with a measurement-based model, and application to the Sioux Falls network also verifies that the proposed CO_2 pricing policy aligned with a more general concept of pricing externality can also reduce the total travel time.

The impacts of different pricing levels on the Sioux Falls network during the peak hour are also investigated. Within the set pricing range, CO_2 pricing reduces the total travel emissions together with the total travel time and increases the revenue while meeting the original demand. The results from numerical experiments also show a monotone relation between price and the changes in the three indicators within the set pricing range.

For future research, there are some potential directions. Although widely used, the static traffic assignment models have some well-known limitations. More efforts should be devoted to moving from this static assignment model to a dynamic assignment model to better achieve time-varying flow assignment. Furthermore, in traffic flow simulation, the deceleration of vehicles at junctions which are not signal-controlled and more factors affecting vehicle movements should also be considered to simulate more realistic vehicle behaviours, reflecting a more accurate relation between flow and vehicle emissions.

Acknowledgements

This work is financially supported by the Natural Science Foundation of Shanghai under grant No. 22ZR1465800 and supported by the Key Laboratory of Road and Traffic Engineering, Ministry of Education.

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