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Design criteria for strut-and-tie modelling in Hong Kong practice

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ABSTRACT

Strut-and-tie modelling has proved a very useful method in the analysis and design of non-flexural components of reinforced concrete members. Examples of design codes encompassing the strut-and-tie model (STM) provisions include Eurocode 2 (EC2), the *fib* Model Code 2010 (MC2010), the Canadian Standard (CSA A23.3-04), the American bridge design specifications (AASHTO LRFD 2020), the American Standard (ACI 318-14 and -19) and the Australian Standard (AS 3600:2018). Nevertheless, the application of different assumptions, model types and methodologies means that the strength acceptance criteria for struts, nodes and bearings vary within the literature and design codes. Unifying the STM design criteria is thus encouraged, to facilitate worldwide use of the STM in the design of non-flexural components, particularly for places such as Hong Kong that have not yet developed the localised design criteria for the STM. In this paper, the proposals set out in the literature and the provisions in the above-stated design codes are reviewed and compared with each other. Calibrating with the existing literature and design provisions, unified STM design criteria for struts, nodes and bearings with localisation are proposed for application in Hong Kong. The proposed design criteria are then applied to the STM design of a cantilever deep beam.

KEYWORDS Strut; tie; node; bearing; non-flexural; deep beam; design; strength
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1. Introduction

The structural analysis and design of concrete members are usually based on the fundamental principle of the Euler-Bernoulli Beam Theory. It is, however, well-known that this conventional beam theory may not apply well to the non-flexural regions of reinforced concrete (RC) structures, such as deep members, pile caps, joints, corbels, nibs and squat walls, where the shear span-depth ratio (*a/d*) is smaller than about 2.5. Strut-and-tie modelling is a proven method that is simple and rational in the analysis and design of these non-flexural components. The strut-and-tie model (STM) is an idealised pin-jointed truss, comprising concrete compression struts, reinforcement tension ties and concrete nodes.

The consistent STM design approach, with consideration of different loading situations, was systematically developed in the 1980s under the efforts of Schlaich and Weischede (1982), Marti (1985), Collins and Mitchell (1986), Rogowsky and MacGregor (1986) and Schlaich et al. (1987). Since then, the STM has rapidly gained in popularity and has been incorporated into various design codes, such as Eurocode 2 (EC2), the *fib* Model Code 2010 (MC2010), the Canadian Standard (CSA A23.3-04), the American bridge design specifications (AASHTO LRFD 2020), the American Standard (ACI 318-14 and -19) and the Australian Standard (AS 3600:2018). Nevertheless, there exist inconsistent sets of strength acceptance criteria for struts, nodes and bearings within the array of design codes and models proposed by various researchers. Thus, unification of the design criteria for the STM is encouraged, to facilitate its worldwide use in the design of non-flexural components.

By reviewing and calibrating with the existing literature and design codes, this paper recommends unified STM design criteria for struts, nodes and bearings. As the current Hong Kong concrete code is largely based on the withdrawn British design standard (BS 8110-1:1997), a comprehensive STM design method is still missing locally. Localised STM design criteria are proposed for practical use in Hong Kong. An example STM design of a cantilever deep beam based on the proposed design criteria is presented for reference.

2. Efficiency factors

According to various design codes and research models, the strength of struts, nodes and bearings can be affected by various factors, such as brittleness effects with increasing concrete strength, amount of confinement, degree of stress disturbance and types of struts, nodes and bearings. To quantify the effects of these factors, the effective strength (f_{ce}) of different STM components can be determined by Equation (1), in which one or multiples of these factors are considered in a single factor β, known as the efficiency factor. The effective strength is normally lower than the compressive cylinder strength of concrete (f_c'), unless under very well-confined conditions.

$$f_{ce} = \beta f_c' \ . \tag{1}$$

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3. STM proposals from the literature

3.1. Strength of struts

Table 1 lists the strength of struts proposed in various studies in terms of the strut efficiency factor (β_{se}). Concrete strength and constant factor were the two most commonly applied models within early studies, such as the strength-dependent factor in Equation (2) proposed by Nielsen et al. (1978) and the 0.6 constant factor proposed by Marti (1985) for general application. The strength models would result in lower efficiency factors as concrete strength increases. Rapid to emerge thereafter were various models based on strain, strut angle and shear span-depth ratio. One of the popular models was the empirical strain-based model proposed by Collins and Mitchell (1986) and Vecchio and Collins (1986), as shown in Equation (3), which was based on their framework of the modified compression field theory (MCFT). The principal tensile strain of concrete (ε_1) is dependent on the strut angle (θ), to account for the effect of strain incompatibility between the strut and the tie. The strut efficiency factor decreases as the strut angle reduces.

Following the compound factor framework of MacGregor (1997), Su and Chandler (2001) established the dual parameters model shown in Equation (4), in which the strength-based partial factor was taken from Model Code 90 and the partial factor in relation to the strut angle was taken from the modified Collins and Mitchell relationship, as transformed by Equation (3).

Tuchscherer et al. (2014) put forward a combined approach to checking the strength of struts, nodes and bearings by considering a strength check at the three faces of a node, namely bearing face, back face and strut-to-node interface, with the single form of equation given in Equation (6). Therein, v is the concrete efficiency factor, which can be the efficiency factor for struts, nodes and bearings, and m is the confinement modification factor, allowing for the effect of concrete confinement when the load distribution area (A_2) is larger than the loaded area (A_1), as defined in Figure 1. m is evaluated as $\sqrt{A_2/A_1} \leq 2$, which is equivalent to that which has been in use for bearing strength calculations in many design codes such as ACI 318 for many years. Equation (5) corresponds to the strength check at the strut-to-node interface of a node.

$$f_{ce} = m v f_c'. \tag{6}$$

Su and Looi (2016) revised the unreinforced strut efficiency factors into the single constant 0.7 based on the experimental study of deep beams. The proposed factor was found to be suitable for high-strength concrete and various strut angles.

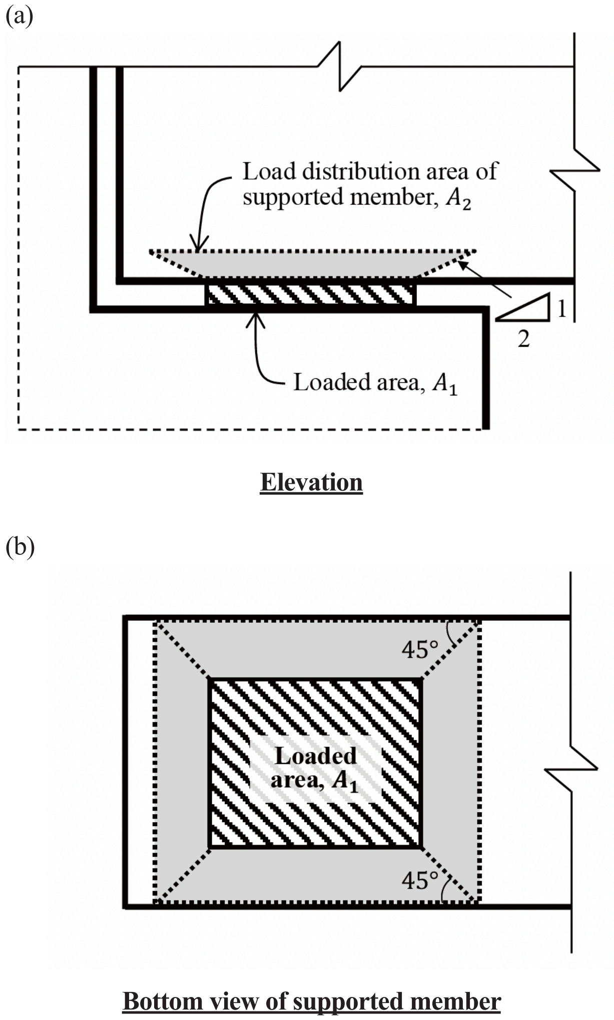

Figure 1. Load distribution area and loaded area for determining the confinement modification factor (adopted in ACI 318).

3.2. Strength of nodes

Table 2 summarises the nodal strength proposed by various researchers in terms of the node efficiency factor (β_{ne}). Marti (1985) adopted the 0.6 constant factor based on past experimental results. Subsequently, this became prevalent in defining nodal strength according to the number of tension ties connected to a node. The common nodal types are CCC, CCT (anchoring one tie) and CTT (anchoring two or more ties), where C represents the compression strut and T denotes the tension tie. The node efficiency factors proposed by Schlaich et al. (1987) and MacGregor (1988) reduce as the number of tension ties and resulting adverse effects increase. Jirsa et al. (1991) conducted tests on CCT and CTT nodes and proposed the 0.8 constant factor.

Table 1. Proposed strut efficiency factors in various studies.

Sources	Strut efficiency factors β_{se}		Eq.
	Equations	Values#	
Nielsen et al. (1978)	$0.7 - f_c'/200$ (for $f_c' \leq 60$ MPa)	0.50	(2)
Marti (1985)	-	0.60	-
Collins and Mitchell (1986) and Vecchio and Collins (1986)	$\frac{1}{0.8+170\varepsilon_1} \leq 0.85$ $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + \varepsilon_2)\cot^2\theta$	0.85, $\theta = 90°$^ 0.73, $\theta = 60°$ 0.55, $\theta = 45°$ 0.31, $\theta = 30°$	(3)
Su and Chandler (2001)	$\beta_{s1}\beta_{s2}$ $\beta_{s1} = 1/(1.14 + 0.75\cot^2\theta)$ $\beta_{s2} = 1.15(1 - f_c'/250)$	0.85, $\theta = 90°$ 0.69, $\theta = 60°$ 0.51, $\theta = 45°$ 0.28, $\theta = 30°$	(4)
Tuchscherer et al. (2014)	$m\beta_s$ $(m \leq 2)$ $\beta_s = 0.85 - f_c'/138$ $(0.45 \leq \beta_s \leq 0.65)$ reinforced $\beta_s = 0.45$ unreinforced	$0.56m \leq 1.12$ $0.45m \leq 0.90$	(5)
Su and Looi (2016)	-	0.70	-

#Assume normal strength concrete, $f_c' = 40$ MPa
^Assume steel yield strain, $\varepsilon_s = 0.002$, and principal compressive strain in strut, $\varepsilon_2 = 0.002$

Table 2. Proposed node efficiency factors in various studies.

Sources	Node efficiency factors β_{ne}				Eq.
	Equations	CCC#	CCT#	CTT#	
Marti (1985)	-		0.60		-
Schlaich et al. (1987)	-	0.85	0.68	/	-
MacGregor (1988)	-	0.85	0.65	0.50	-
Jirsa et al. (1991)	-	/	0.80	0.80	-
MacGregor (1997)	$\beta_{n1}\beta_{n2}$ $\beta_{n1} = 1.00$ CCC $\beta_{n1} = 0.85$ CCT $\beta_{n1} = 0.75$ CTT $\beta_{n2} = 0.55 + 1.25/\sqrt{f_c'}$	0.75	0.64	0.56	(7)
Su and Chandler (2001)	$\beta_{n1}\beta_{n2}$ $\beta_{n1} = 0.85$ CCC $\beta_{n1} = 0.75$ CCT $\beta_{n1} = 0.65$ CTT $\beta_{n2} = 1.15(1 - f_c'/250)$	0.82	0.72	0.63	(8)
Tuchscherer et al. (2014)	$m\beta_n$ $(m \leq 2)$ $\beta_n = 0.85$ CCC $\beta_n = 0.70$ CCT bearing face	$0.85m$	$0.70m$	/	(9)
Typical values (for $m = 1$)		0.85	0.70	0.60	-

#Assume normal strength concrete, $f_c' = 40$ MPa

Table 3. Proposed bearing strength in various studies.

Sources	Bearing strength		Remarks	Eq.
	Equations	Values# (in terms of f'_c)		
Hawkins (1968)	$\left[1+\frac{4.15}{\sqrt{f'_c}}\left(\sqrt{A_2/A_1}-1\right)\right]f'_c$	2.31, $m = 3$ 1.66, $m = 2$ 1.00, $m = 1$	Ultimate strength	(10)
Schlaich et al. (1987)	$0.6f'_c$	0.60	Uncracked strength	-
Adebar and Zhou (1993)	$0.6(1+2\alpha_b\beta_b)f'_c$ $\alpha_b = 0.33\left(\sqrt{A_2/A_1}-1\right) \leq 1$ $\beta_b = 0.33(h/b-1) \leq 1$		Uncracked strength (increasing 83% becomes the ultimate strength)	(11)
	For $f'_c > 34.5$ MPa, $\left[0.6+\frac{6}{\sqrt{f'_c}}(\alpha_b\beta_b)\right]f'_c$	1.23, $m = 3$* 0.91, $m = 2$ 0.60, $m = 1$		(12)
Su and Chandler (2001)	1.30 × Equations proposed by Adebar and Zhou (1993)	1.59, $m = 3$* 1.19, $m = 2$ 0.78, $m = 1$	Ultimate strength	-
Tuchscherer et al. (2014)	$m\beta_b f'_c\ (m \leq 2)$ $\beta_b = 0.85$ bearing face of CCC node $\beta_b = 0.70$ bearing face of CCT node	 $0.85m \leq 1.70$ $0.70m \leq 1.40$	Ultimate strength	(13)

Assume normal strength concrete, $f'_c = 40$ MPa

* Assume aspect ratio (height/width) of strut, $h/b \geq 4$ such that $\beta_b = 1$

Table 4. Partial safety factors for materials.

Design codes	Concrete		Steel reinforcement	
	Material FOS γ_c	Strength reduction factors ϕ*	Material FOS γ_s	Strength reduction factors ϕ*
HKCP 2013	1.5	0.67	1.15	0.87
EC2 (BS EN 1992-1-1:2004)	1.5	0.67	1.15	0.87
MC2010	1.5	0.67	1.15	0.87
CSA A23.3-04	/	0.65	/	0.85
AASHTO LRFD 2020	/	0.70 for the STM 0.70 for bearings on concrete	/	0.90 for the STM
ACI 318-14 and -19	/	0.75 for the STM 0.65 for bearings on concrete	/	0.75 for the STM
AS 3600:2018	/	0.65 for the STM 0.60 for bearings on concrete	/	0.85 for the STM

* $\phi = 1/\gamma_c$ or $1/\gamma_s$ for HKCP 2013, EC2 and MC2010

MacGregor (1997) proposed the compound factor approach to evaluate the efficiency factor, which is a function of the nodal type and concrete strength, as shown in Equation (7). Based on the compound factor approach, Su and Chandler (2001) adopted typical values from past research and design codes to account for the nodal type as one of the partial factors, and made use of the strength-based factor adopted in Model Code 90 as another partial factor, in Equation (8). Equation (9) is part of the combined approach for checking the strength of nodes proposed by Tuchscherer et al. (2014), who suggested that checking at CTT nodes and the back face of CCT nodes is not necessary provided that the main reinforcement is properly anchored.

In general, without considering the confinement effect, the nodal strengths are more consistent for CCC nodes and CCT nodes, with typical node efficiency factors of 0.85 and 0.7 respectively. For CTT nodes, due to the deficiency of studies, there is more variation in the proposed node efficiency factors, ranging from 0.5 to 0.8 with a typical value of 0.6.

3.3. Strength of bearings

Table 3 lists the bearing strengths proposed in various studies and provides examples of the numerical values in terms of f_c'. Hawkins (1968) proposed Equation (10) based on bearing strength tests on concrete, with the use of rigid plates under different A_2/A_1 ratios. The confinement effect is manifested in the rear term of the equation with a square root relationship of A_2/A_1. Use of the square root relationship of A_2/A_1 (i.e., m) for evaluating bearing strength has since become prevalent among researchers and within various design codes.

Schlaich et al. (1987) proposed the 0.6 constant coefficient of bearing strength to preclude the transverse splitting failure of unreinforced struts under a biaxial stress state. By conducting analytical studies and carrying out an experimental study of the bearing strength of concrete cylinders, Adebar and Zhou (1993) proposed Equations (11) and (12) to evaluate the uncracked bearing strength for plain concrete. Su and Chandler (2001) established ultimate strength values based on the uncracked strength equations established by Adebar and Zhou (1993). A conservatively assumed multiplication factor of 1.3 was chosen by Su and Chandler (2001) empirically for the conversion, although the experiments carried out by Adebar and Zhou (1993) found an average 83% increase from uncracked strength to ultimate strength. Equation (13) is the bearing strength proposal extracted from the combined strength checking approach by Tuchscherer et al. (2014).

4. STM provisions in design codes

In Hong Kong, although there is no detailed guidance on the STM design approach, bearing strength requirements for concrete structures are specified in the Hong Kong code (Code of Practice for Precast Concrete Construction 2016 (HKCPPCC 2016)) with fundamental structural design principles specified within the Code of Practice for Structural Use of Concrete 2013 (HKCP 2013). The local bearing strength requirement will be reviewed and compared with those contained within other international design codes.

4.1. Partial safety factors for materials and loads

To ensure that the strength of each STM component is compared on the same basis among the design codes, the partial safety factors for materials and loads are considered in evaluating design strength. In terms of the material factor of safety (FOS) summarised in Table 4, the effects of the partial safety factors for concrete (γ_c) and steel reinforcement (γ_s) specified in HKCP 2013, EC2, MC2010 may be considered by applying the strength reduction factors $\phi = 1/\gamma_c$ or $1/\gamma_s$ directly to the codified strength, as in other design codes. In AASHTO LRFD 2020, ACI 318, AS 3600:2018, there are provisions regarding the strength reduction factors, particularly for the design using STM. For these cases, there are also separate strength reduction factors defined for the bearing strength on concrete for general application, and it is noticed that the factors for STM and bearings on concrete are different in ACI 318 and AS 3600:2018.

Table 5 lists the partial safety factors for the basic load case (dead load (D) + live load (L)). For ease of comparison, the load factors for dead loads and live loads are combined into an equivalent load factor by assuming that the live load is 25% of the dead load. The load adjustment factors (μ) are normalised, calculated by taking the combined load factor of 1.725 in EC2 and MC2010 as the reference base. By applying the adjustment factors to the strength of each STM component, the load factors can be implicitly accounted for in the design strength.

Table 5. Partial safety factors for loads.

Design codes	**Load factors: dead load (D) + live load (L)**	**Combined load factors***	**Load adjustment factors μ**
HKCP 2013	1.4D + 1.6L	1.800	0.958
EC2 as per BS EN 1990:2002	1.35D + 1.5L	1.725	1.000
MC2010	1.35D + 1.5L	1.725	1.000
CSA A23.3-04	1.25D + 1.5L	1.625	1.062
AASHTO LRFD 2020	1.25D + 1.75L	1.688	1.022
ACI 318-14 and -19	1.2D + 1.6L	1.600	1.078
AS 3600:2018 as per AS/NZS 1170.0:2002	1.2D + 1.5L	1.575	1.095

* Assume L/D = 0.25

Table 6. Codified strength of struts.

Design codes	Strength reduction factors ϕ	Codified strength of struts	Adjusted design strength[#&]
EC2	0.67	$\phi 0.85(1 - f_c'/250)f_c'$ uniaxial $0.6\phi 0.85(1 - f_c'/250)f_c'$ transverse tension	$0.48f_c'$ $0.29f_c'$
MC2010	0.67	$\phi 0.85\beta_s f_c'$ $\beta_s = 1.0\eta_{fc}$ uniaxial $\beta_s = 0.75\eta_{fc}$ transverse tension $\beta_s = 0.55\eta_{fc}$ oblique tension $\eta_{fc} = (30/f_c')^{1/3} \leq 1.0$	 $0.52f_c'$ $0.39f_c'$ $0.28f_c'$
CSA A23.3-04	0.65	$\phi \dfrac{f_c'}{0.8 + 170\varepsilon_1} \leq 0.85\phi f_c'$ $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)\cot^2\theta$	$0.59f_c'$, $\theta = 90°$^ $0.50f_c'$, $\theta = 60°$ $0.38f_c'$, $\theta = 45°$ $0.22f_c'$, $\theta = 30°$
AASHTO LRFD 2020	0.70	At strut-to-node interface, $\phi m\beta_s f_c'$ $(m \leq 2)$ $\beta_s = 0.85 - f_c'/138$ $(0.45 \leq \beta_s \leq 0.65)$ reinforced $\beta_s = 0.45$ unreinforced	 $0.40mf_c' \leq 0.80f_c'$ $0.32mf_c' \leq 0.64f_c'$
ACI 318-14	0.75	$\phi 0.85\beta_s f_c'$ $\beta_s =$ 1.0 prismatic struts $\beta_s =$ 0.75 reinforced bottle-shaped struts $\beta_s =$ 0.60λ unreinforced bottle-shaped struts* $\beta_s =$ 0.40 struts in tension zones $\beta_s =$ 0.60λ other cases*	 $0.69f_c'$ $0.52f_c'$ $0.41f_c'$ $0.27f_c'$ $0.41f_c'$
ACI 318-19	0.75	$\phi 0.85m\beta_s f_c'$ $(m \leq 2)$ $\beta_s =$ 1.0 boundary struts $\beta_s =$ 0.75 reinforced interior struts, unreinforced interior struts but with sufficient diagonal tensile strength, or located at beam-column joints $\beta_s =$ 0.40 struts in tension zones and interior struts of other cases	 $0.69mf_c' \leq 1.37f_c'$ $0.52mf_c' \leq 1.03f_c'$ $0.27mf_c' \leq 0.55f_c'$
AS 3600:2018	0.65	$\phi 0.9\beta_s f_c'$ $\beta_s =$ 1.0 prismatic struts $\beta_s = \frac{1}{1.0+0.66\cot^2\theta}$ $(0.3 \leq \beta_s \leq 1.0)$ fan-shaped and bottle-shaped struts	 $0.64f_c'$ $0.64f_c'$, $\theta = 90°$ $0.53f_c'$, $\theta = 60°$ $0.39f_c'$, $\theta = 45°$ $0.22f_c'$, $\theta = 30°$

[#] Assume normal weight and normal strength concrete, $f_c' = 40$ MPa
[&] Load adjustment factors μ incorporated
^ Assume steel yield strain, $\varepsilon_s = 0.002$
* λ is a correction factor for lightweight concrete

4.2. Codified strength of struts

Table 6 summarises the strength of struts specified in different design codes. EC2 and MC2010 contain similar provisions, in which the strut efficiency factor is a function of concrete strength and is dependent on the stress state. The 0.85 factor in the expressions accounts for the adverse effect due to sustained loads. This reduction factor is the same as the 0.85 uncertainty factor adopted in ACI 318.

CSA A23.3-04 adopts the strain-based model from Collins and Mitchell (1986) and Vecchio and Collins (1986), which is also the previous approach adopted in AASHTO LRFD. However, since the 2017 edition, AASHTO LRFD has adopted the combined approach in checking the strength of struts, nodes and bearings, as proposed by Tuchscherer et al. (2014). Unlike the proposal from Tuchscherer et al. (2014), strength checking at the bearing face, back face and strut-to-node interface is necessary for all nodal types under AASHTO LRFD 2020.

ACI 318-14 and AS 3600:2018 classify the struts into different geometries based on stress disturbance in evaluating the strength of struts. Recently, there has been

Table 7. Codified strength of nodes.

Design codes	Strength reduction factors ϕ	Codified strength of nodes	Adjusted design strength (in terms of f_c')[#&]		
			CCC	CCT	CTT
EC2	0.67	$\phi 0.85\beta_n(1-f_c'/250)f_c'$ β_n = 1.0 CCC β_n = 0.85 CCT β_n = 0.75 CTT	0.48	0.41	0.36
MC2010	0.67	$\phi 0.85\beta_n f_c'$ $\beta_n = 1.0\eta_{fc}$ CCC $\beta_n = 0.75\eta_{fc}$ CCT & CTT $\eta_{fc} = (30/f_c')^{1/3} \leq 1.0$	0.52	0.39	0.39
CSA A23.3-04	0.65	$\phi 0.85 f_c'$ CCC $\phi 0.75 f_c'$ CCT $\phi 0.65 f_c'$ CTT	0.58	0.52	0.45
AASHTO LRFD 2020	0.70	At bearing face and back face, $\phi m\beta_n f_c'$ $(m \leq 2)$ β_n = 0.85 CCC β_n = 0.70 CCT $\beta_n = 0.85 - f_c'/138$ CTT $(0.45 \leq \beta_n \leq 0.65)$ reinforced $\beta_n = 0.45$ CTT unreinforced	0.61m	0.50m	0.40m (reinforced) 0.32m (unreinforced)
ACI 318-14	0.75	$\phi 0.85\beta_n f_c'$ β_n = 1.0 CCC β_n = 0.80 CCT β_n = 0.60 CTT	0.69	0.55	0.41
ACI 318-19	0.75	$\phi 0.85 m\beta_n f_c'$ $(m \leq 2)$ β_n = 1.0 CCC β_n = 0.80 CCT β_n = 0.60 CTT	0.69m	0.55m	0.41m
AS 3600:2018	0.65	$\phi 0.9\beta_n f_c'$ β_n = 1.0 CCC β_n = 0.80 CCT β_n = 0.60 CTT	0.64	0.51	0.38
Typical values (for m = 1)			0.60	0.50	0.40

[#] Assume normal strength concrete, f_c' = 40 MPa
[&] Load adjustment factors μ incorporated

a major revision of the STM provision in the American Standard (ACI 318-19). The strut efficiency factors have been revised for the new definitions of struts in terms of boundary struts and interior struts, and to preclude diagonal tension failure for interior struts. As in the case of AASHTO LRFD 2020, the confinement modification factor is allowed to account for the possible increment in effective strength under ACI 318-19.

In short, the design strength estimated using EC2 or MC2010 is relatively conservative when compared with the others. Without considering the confinement effect, prismatic struts, boundary struts and 90° angle struts possess the highest adjusted design strength, ranging from $0.48f_c'$ to $0.69f_c'$. With an increasing degree of stress disturbance, the design strength of bottle-shaped struts, interior struts and inclined struts reduces to between $0.29f_c'$ and $0.53f_c'$. For struts in tension zones or inclined at relatively shallow angles of 30°, the strengths are the lowest.

4.3. Codified strength of nodes

Table 7 presents nodal strengths codified in accordance with various design codes. EC2 and MC2010 adopt a similar form of strength equation for struts when calculating nodal strength, considering the brittleness effect of higher-strength concrete. Similar to the strength of struts being evaluated at the strut-to-node interface, the confinement effect is considered when calculating nodal strength in AASHTO LRFD 2020 and ACI 318-19. In general, if the confinement effect is ignored, the nodal strengths suggested in EC2 and MC2010 are more conservative than the others. The consistency of strength is the highest for CTT nodes with a typical design strength of $0.4f_c'$ while the strength varies more for CCC and CCT nodes. The typical design strengths of CCC nodes and CCT nodes are $0.6f_c'$ and $0.5f_c'$ respectively.

Table 8. Codified bearing strength.

Design codes	Strength reduction factors ϕ	Codified bearing strength	Adjusted design strength[&]
HKCPPCC 2016	0.67	$0.4f_{cu}$ dry bearing $0.6f_{cu}$ bedded bearing $0.8f_{cu}$ cast-in steel plate	$0.48f_c'$ $(0.41f_c')^{\#*}$ $0.72f_c'$ $(0.61f_c')$ $0.96f_c'$ $(0.81f_c')$
EC2 and MC2010	0.67	$0.4\phi 0.85f_c'$ dry bearing $0.85\phi 0.85f_c'$ other bedding cases $\phi 0.85mf_c'$ $(m \leq 3)$ partially loaded areas	$0.23f_c'$ $0.48f_c'$ $0.57mf_c' \leq 1.71f_c'$
CSA A23.3-04	0.65	$\phi 0.85mf_c'$ $(m \leq 2)$	$0.59mf_c' \leq 1.17f_c'$
AASHTO LRFD 2020	0.70	$\phi 0.85\beta_b f_c'$ $\beta_b = m \leq 2$ uniform stresses $\beta_b = 0.75m \leq 1.5$ non-uniform stresses	 $0.61mf_c' \leq 1.22f_c'$ $0.46mf_c' \leq 0.91f_c'$
ACI 318-14 and -19	0.65	$\phi 0.85mf_c'$ $(m \leq 2)$	$0.60mf_c' \leq 1.19f_c'$
AS 3600:2018	0.60	$\phi 0.9mf_c'$ $(m \leq 2)$	$0.61mf_c' \leq 1.23f_c'$

[&] Load adjustment factors μ incorporated
[#] Assume $f_c'/f_{cu} = 0.8$ for converting cube strength into cylinder strength
* 0.85 uncertainty factor is included for the values in the parentheses

4.4. Codified strength of bearings

Table 8 lists the bearing strength provisions specified in different design codes. Most design codes make use of the simplified Hawkins's expression in the form of mf_c' to account for the confinement effect. The value of m is limited to 2 in most design codes, aside from the more lenient value of 3 specified in EC2 and MC2010.

HKCPPCC 2016, EC2 and MC2010 classify bearing strength based on bedding conditions. HKCPPCC 2016 specifies three types of bedding conditions, namely dry bearing, bedded bearing and cast-in steel plate. Dry bearing refers to a scenario without a bedded material, such that there is a non-uniform stress distribution at the bearing surface. For bedded bearing, the uniform stress distribution is anticipated, enabling a higher bearing strength. A further enhancement in bearing strength is possible under cast-in steel plate conditions with sufficient confinement. EC2 and MC2010 adopt a similar bearing strength classification to HKCPPCC 2016 except that the cast-in steel plate conditions are not specifically considered, and the confinement effect is expressly considered for partially loaded conditions.

To compare the bearing strength on the same basis, a further conversion was made to HKCPPCC 2016 from cube strength (f_{cu}) into cylinder strength (f_c') by assuming f_c'/f_{cu} = 0.8 for the adjusted design strength. If the 0.85 uncertainty factor is included in HKCPPCC 2016, the design strength for bedded bearing would become $0.61f_c'$, which is roughly consistent with that of the partially loaded conditions in EC2 and MC2010, without considering the confinement effect. This value is also similar to that of the remaining design codes. As such, the bearing strength under bedded bearing in the existing Hong Kong code may correspond equivalently to the bearing strength provisions using the simplified Hawkins's expression in other design codes, which all refer to the uniform stress condition.

5. Geometric definition of nodal region

The proposed strengths of different STM components vary throughout the literature and design codes. One potential contributory reason for this is the geometric definition of the nodal region.

5.1. Effect of hydrostatic and non-hydrostatic nodal conditions

A node can be classified as either a hydrostatic node or a non-hydrostatic node, as illustrated in Figure 2. For hydrostatic nodes, the applied forces are normal to each nodal face with equal stresses. Correspondingly, there is no shear stress across the entire node, and the lengths of the nodal faces are in the same proportions as the respective forces. However, hydrostatic nodes may not always reflect the realistic geometry of nodes and struts or the practical reinforcement arrangement, particularly for excessive nodal depths resulting from high shear span-depth ratios. To eliminate such deficiency, non-hydrostatic nodes wherein forces are not always perpendicular to the nodal faces can be adopted to allow greater flexibility in sizing the nodes.

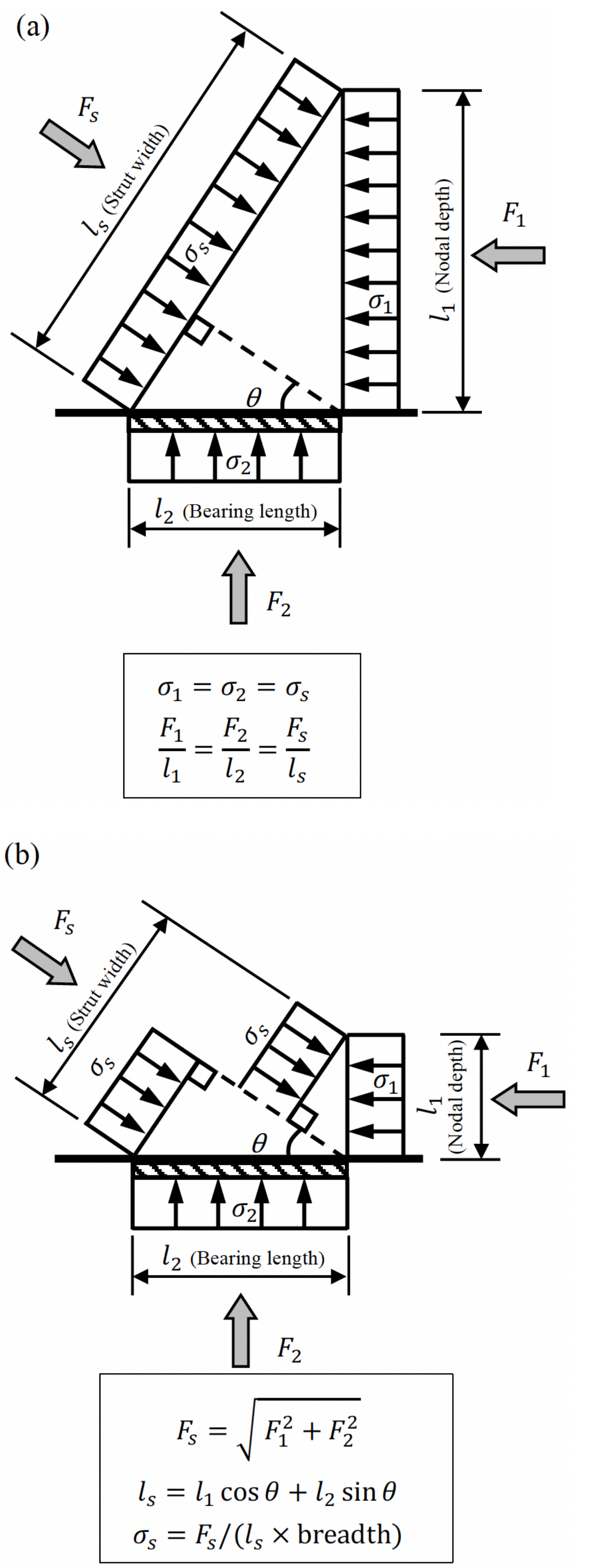

Figure 2. Nodal type: (a) hydrostatic node; and (b) non-hydrostatic node.

Tuchscherer et al. (2016) demonstrated the importance of choosing the appropriate strut efficiency factor for non-hydrostatic nodes by comparing the shear strength provisions in ACI 318-11 (equivalent to ACI 318-14) and the previous AASHTO LRFD (equivalent to CSA A23.3-04) with the combined approach in Equation (6) proposed by Tuchscherer et al. (2014). The design strengths of struts from CSA A23.3-04 in Table 6 are relatively conservative at small strut angles as compared with the provisions in ACI 318 and the factored strengths evaluated from Equation (6). Tuchscherer et al. (2016) considered that, when such a low strength provision (which is likely more suitable for the hydrostatic nodal condition) is applied to the non-hydrostatic node at high shear span-depth ratios, excessive conservatism in relation to shear resistance may result due to the small strut width evaluation. As such, the strain-based approach from CSA A23.3-04 and other similar proposals based on the MCFT are not recommended for evaluating the strength of struts under the non-hydrostatic nodal condition.

5.2. Effect of depth of compression zone

Within the context of deep beams, the depth of compression zone refers essentially to the depth of CCC node. Varying the depth of CCC node results in different strut widths. For shear strength controlled by the strength of a CCC node at its strut-to-node interface, the difference in strut widths would imply different strut efficiency factors for developing the same shear strength. As such, the CCC nodal depth should be unified for evaluation of the strength of struts. As illustrated in Figure 3, there are three commonly adopted definitions for CCC nodal depths (h_{CCC}), including (Definition 1) the depth of the equivalent compressive stress block from conventional flexural analysis, (Definition 2) the depth of the linear elastic stress distribution from elastic theory and (Definition 3) the depth based on the static equilibrium of the STM system at shear failure and plastic stress state, which corresponds to the intermediate stress levels between Definitions 1 and 2.

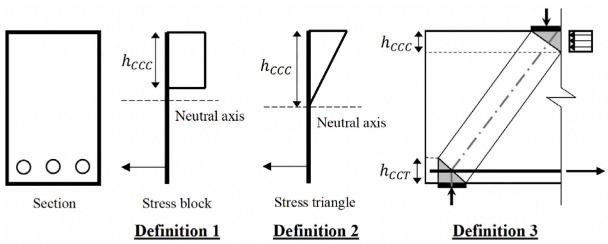

Figure 3. Three commonly adopted definitions of depth of compression zone.

An investigation into the effect of CCC nodal depth definitions on the strut efficiency factor is carried out hereafter, based on the experimental study by Su and Looi (2016), whose adopted STM configuration is shown in Figure 4. Su and Looi (2016) purposely designed the deep beams to have strength control at the strut-to-node interface of the CCC node. Three target concrete strengths f_c' in a matrix of various target shear span-moment arm ratios (*a/z*) of 0.5, 1.0 and 1.7 were designed: 30 MPa, 60 MPa and 90 MPa. Based on Definition 3 of the nodal depth and non-hydrostatic nodal condition, they proposed the 0.7 nominal strut efficiency factor.

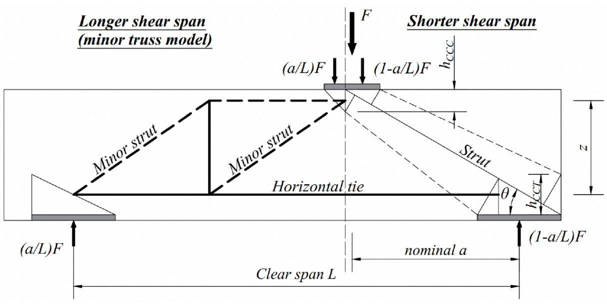

Figure 4. STM configuration adopted in an experimental study by Su and Looi (2016).

Figure 5 shows the strut efficiency factors recalculated against the *a/z* ratio for different definitions of the CCC nodal depth. For each *a/z* ratio, there are three data points for each nodal depth definition, corresponding to the three target concrete strengths. Compared with the 0.7 strut efficiency factor recommended by Su and Looi (2016) using Definition 3, lower efficiency factors of 0.6 and 0.4 are needed to be considered sufficiently safe for Definitions 1 and 2 respectively. For Definition 3, the CCC nodal depth, and hence the strut width, is the smallest of the three, meaning that higher efficiency factors are sufficient to maintain the same level of safety. As Definition 3 corresponds to the intermediate stress levels achieved at plastic shear failure, which are often the cases in occurrence, it is recommended that Definition 3 be adopted for CCC nodal depth and the corresponding strut efficiency factor in the STM design.

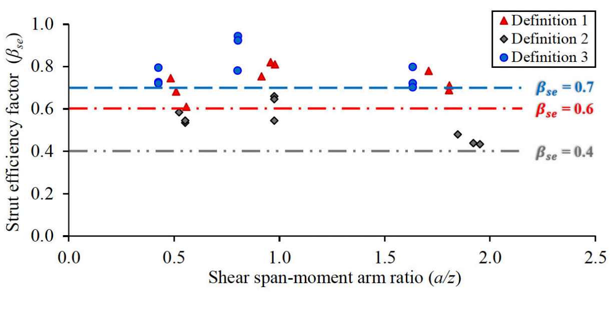

Figure 5. Strut efficiency factors under different definitions of CCC nodal depths using experimental results from Su and Looi (2016).

6. Recommended STM design acceptance criteria

Considering the strength of struts, the 0.7 nominal strut efficiency factor proposed by Su and Looi (2016) is recommended as it has been calculated under the non-hydrostatic nodal condition and uses Definition 3 for the CCC nodal depth, with considerations of different strut angles and concrete strengths. Su and Looi (2016) suggested including the 0.85 uncertainty factor in the strength of struts (as in ACI 318) to account for the long-term load effects, such that the strut efficiency factor would become 0.6. Further, taking the 1.5 partial safety factor for concrete into account, a design strut strength of $0.4f_c'$ results. In order to align with the trend of the two recent design codes, AASHTO LRFD 2020 and ACI 318-19, the confinement modification factor ($m = \sqrt{A_2/A_1} \leq 2$) is included.

For the strength of nodes, with the moderately high consistency of the strength provisions in various design codes (as given in Table 7), the typical values of $0.50f_c'$ and $0.40f_c'$ are recommended as the design strengths for CCT and CTT nodes respectively. For CCC nodes, the typical strength of $0.85f_c'$ (as given in Table 2) has been suggested by many researchers to be the limiting value and hence such value is also recommended for local application. By incorporating the 1.5 partial safety factor for concrete, a factored strength of $0.56f_c'$ is taken for CCC nodes. As with the strength of struts, the confinement effect is considered by adopting the confinement modification factor.

Regarding the strength of bearings, a more conservative unfactored bearing strength is proposed herein as $0.75mf_c'$ for bedded bearings as compared with the $0.85mf_c'$ in ACI 318. It is emphasised that the proposed bearing strength is essentially the same as the design strength of CCT nodes ($0.50mf_c'$ factored strength) as recommended previously. Such a scenario is consistent with typical cases of member supports where the concrete nodes under consideration are the CCT nodes in the context of the STM.

In Figure 6, the proposed bearing strength is compared with the unfactored provisions in ACI 318 and HKCPPCC 2016 and the proposals in various studies. It has been considered by numerous researchers such as Adebar and Zhou (1993) that ACI 318 may not be conservative, particularly for high-strength concrete. In the case of the ultimate bearing strengths evaluated by Adebar and Zhou (1993) and Hawkins (1968) for $f_c' = 34.5$ MPa, they possess the same level of conservatism as the provision in ACI 318 at approximately $m = 2$. As the concrete strength increases beyond 34.5 MPa, the calculated unfactored bearing strength using ACI 318 becomes unconservative for m as it reaches 2.

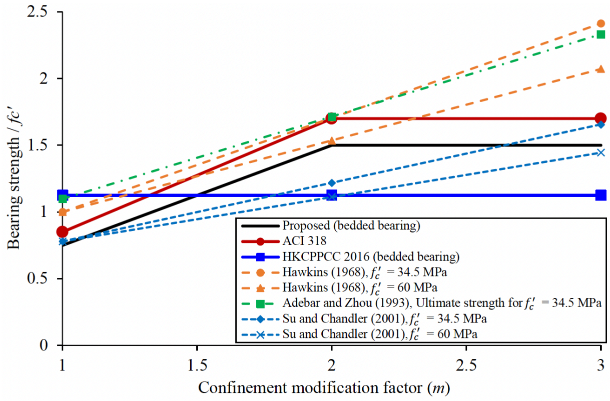

Figure 6. Comparison of proposed bearing strength with various models.

By using the proposed bearing strength, a moderate conservatism can be maintained, as compared with the proposal from Hawkins (1968) for high-strength concrete f_c' = 60 MPa at m = 2. On the other hand, excessive conservatism is avoided, with the proposed bearing strength being higher than the lower bound of the ultimate bearing strength from Su and Chandler (2001) for intermediate m. In addition, the proposed bearing strength exhibits improvement over the provision in HKCPPCC 2016. It not only tackles the issue of obvious unconservative strength prediction under low confinement for the existing Hong Kong code but can also align with the global trend of taking the benefit of the confinement effect.

Table 9 summarises the proposed design strength of struts, nodes and bearings for the STM. The current bearing strength provisions in the Hong Kong code are adhered to for both dry bearing and cast-in steel plate conditions, having correlated with bedded bearings. For application in Hong Kong, a conversion from cylinder strength into cube strength is made by assuming $f_c'/f_{cu} = 0.8$.

7. Minimum distributed reinforcement

Although the proposed design strength of struts can be applied to members without sufficient distributed reinforcement, as demonstrated in the experiment by Su and Looi (2016), minimum reinforcement is recommended for both crack control and strength. As the current provisions concerning distributed reinforcement in HKCP 2013 are not tailored to the STM, a minimum distributed reinforcement of 0.25% in the form of an orthogonal grid placed evenly at each face of the section is recommended with reference to the typical 0.2% to 0.3% proposals in the literature and design code provisions. The proposed 0.25% minimum distributed reinforcement is consistent with the latest provision in ACI 318-19.

8. Design example of a cantilever deep beam

A design example of a cantilever deep beam is presented below to demonstrate the STM design procedure, based on the proposed design criteria presented in Table 9. Figure 7 shows the cantilever deep beam under an ultimate point load of 18000 kN. The bearing plate is assumed to be rigid such that uniform stress distribution exists as with bedded bearings. The characteristic compressive cube strength of concrete (f_{cu}) and the yield strength of steel reinforcement (f_y) are 45 MPa and 500 MPa respectively. For simplicity, the weight of the member itself is ignored.

Table 9. Summary of proposed STM design strength.

Components	Type	Design strength: Cylinder strength	Design strength: Cube strength	Remarks
Strut	/	$0.40mf_c'$	$0.32mf_{cu}$	Value from Su and Looi (2016).
Node	CCC	$0.56mf_c'$	$0.45mf_{cu}$	Typical values from the literature and design codes. The 0.45 factor for the CCC node is consistent with that of the maximum concrete compressive stress adopted in HKCP 2013.
	CCT	$0.50mf_c'$	$0.40mf_{cu}$	
	CTT	$0.40mf_c'$	$0.32mf_{cu}$	
Bearing	Dry bearing	$0.34mf_c'$	$0.27mf_{cu}$	In line with the provision in HKCPPCC 2016 taking 2/3 of the value for bedded bearings.
	Bedded bearing	$0.50mf_c'$	$0.40mf_{cu}$	Consistent with the CCT nodal strength.
	Cast-in steel plate	$1.00f_c'$	$0.80f_{cu}$	Consistent with the value for bedded bearings under full confinement.

Note 1: A partial safety factor for concrete of 1.5 is allowed

Note 2: $m = \sqrt{A_2/A_1} \leq 2$

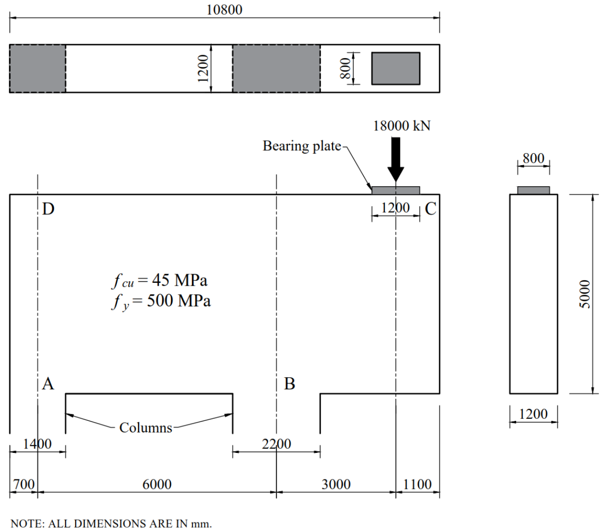

Figure 7. Cantilever deep beam for design using the STM.

8.1. Step 1: Identification of non-flexural components (D-regions)

The non-flexural regions of members are regarded as D-regions (D standing for disturbed or discontinuity) in the context of the STM. The extent of each D-region can be found by St. Venant's Principle, which suggests that the local stress concentration caused by a disturbance smooths out to the uniform stress distribution within a distance that is sufficiently distant from the discontinuity, and this distance is taken as the member depth. Locations outside the D-regions are B-regions (B standing for Bernoulli or Beam), where conventional flexural beam theory can be applied suitably. Based on St. Venant's Principle, the whole member is regarded as the D-region since all locations in the member are within the member depth (i.e., 5 m from the applied load or the support reaction forces).

8.2. Step 2: Evaluation of boundary forces/stresses

Considering static equilibrium, the support reactions at the bearing faces of nodes A and B are determined as 9000 kN (downward) and 27000 kN (upward) respectively. Where the bearing faces at nodes B and C are under compression, the nodal or bearing stresses are first checked against the corresponding nodal or bearing strengths. For node B (CCC node), the nodal strength is 0.45 x 45 = 20.3 MPa, which is greater than the stress at the bearing face of 27000 x 10^3 / (2200 x 1200) = 10.2 MPa. Meanwhile, for node C (CCT node), with a confinement modification factor (m) of $\sqrt{(1600\times1200)/(1200\times800)}=1.414$, the nodal/bearing strength is determined as 0.40 x 1.414 x 45 = 25.5 MPa, which is greater than the bearing stress of 18000 x 10^3 / (1200 x 800) = 18.8 MPa.

8.3. Step 3: Development and dimensioning of the STM

Figure 8 shows the STM idealised for the cantilever deep beam. The computed dimensions of nodes, struts and internal forces are to be explained. To begin, node B is divided into two sub-nodes for the transfer of forces at strut-to-node interfaces with struts BC and BD respectively. For the same amount of bearing stress, the bearing lengths at the two sub-nodes are dimensioned according to the ratio of the two shear spans, which are 3000 mm and 6000 mm. Based on the resulting ratio of 1 to 2, the bearing length at node B for the strut BD side is calculated as 733 mm and that for the strut BC side is determined as 1467 mm.

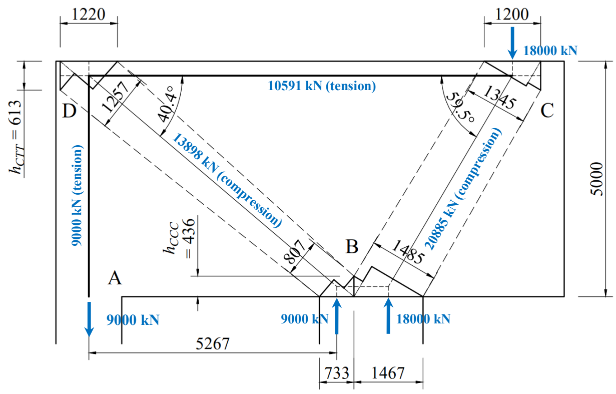

Figure 8. Dimensioning of the STM for the cantilever deep beam.

Thereafter, the two nodal depths h_{CCC} and h_{CTT} are found based on the equilibrium of the STM system at plastic shear failure. Considering the free body diagram of the longer shear span side, and by limiting the nodal stresses of node B (CCC node) and node D (CTT node) to the corresponding design strengths, the relationship between the two nodal depths can be established based on the force and moment equilibriums. For horizontal force equilibrium between the allowable compression at B and the allowable tie CD tension at node D, $0.45f_{cu}h_{CCC} = 0.32f_{cu}h_{CTT}$ such that $h_{CTT} = 1.406h_{CCC}$. The solutions can be found by substituting $h_{CTT} = 1.406h_{CCC}$ into the following moment equilibrium equation:

$$0.45f_{cu}(h_{CCC}\times 1200)\left(5000-\frac{h_{CCC}}{2}-\frac{h_{CTT}}{2}\right)=9000\times10^3\times5267. \quad (14)$$

Having solved the above equation, the minimum values of h_{CCC} and h_{CTT} are found to be 436 mm and 613 mm respectively. With the calculated nodal depths, the internal forces of struts and ties and all other dimensions can be found as follows (except that the bearing length of 1220 mm at node D is to be determined in Step 4):

Tie CD force: $0.45 \times 45 \times 436 \times 1200 \times 10^{-3} = 10591$ kN [horizontal compression at B]

Strut BC force: $\sqrt{10591^2 + 18000^2} = 20885$ kN [equilibrium at node C]

Strut BD force: $\sqrt{10591^2 + 9000^2} = 13898$ kN [equilibrium at node D]

Strut BC angle: $\tan^{-1}(18000/10591) = 59.5°$ [equilibrium at node C]

Strut BD angle: $\tan^{-1}(9000/10591) = 40.4°$ [equilibrium at node D]

Strut BC width at node B: $436 \cos 59.5° + 1467 \sin 59.5° = 1485$ mm [non-hydrostatic node]

Strut BC width at node C: $613 \cos 59.5° + 1200 \sin 59.5° = 1345$ mm [non-hydrostatic node]

Strut BD width at node B: $436 \cos 40.4° + 733 \sin 40.4° = 807$ mm [non-hydrostatic node]

Strut BD width at node D: $613 \cos 40.4° + 1220 \sin 40.4° = 1257$ mm [non-hydrostatic node]

8.4. Step 4: Determination of tie reinforcement

With a design strength of $0.87f_y$ under HKCP 2013, the required steel area for tie CD is 10591 x 10^3 / (0.87 x 500) = 24346 mm^2. Accordingly, 8T32 in four layers with a total steel area of 25736 mm^2 is provided, and each layer of reinforcement is distributed at 200 mm c/c with a total depth of more than h_{CTT} = 613 mm (say 750 mm). Referring to clause 8.4.5 of HKCP 2013, the required anchorage length at nodes C and D is 33ϕ = 1056 mm. The bearing length of 1200 mm for node C and 1220 mm for node D are sufficient for the proper transfer of bond forces within the two nodal regions.

For tie AD, the required steel area is 9000 x 10^3 / (0.87 x 500) = 20690 mm^2. Accordingly, 8T20 in nine layers with a total steel area of 22619 mm^2 is provided, and each layer of reinforcement is distributed at 150 mm c/c, leading to a minimum 1220 mm bearing length at node D. The required anchorage length at node D is 33ϕ = 660 mm, which is less than the available length for anchorage of 750 – 40cover = 710 mm.

8.5. Step 5: Strength checks for nodes and struts

Node B (CCC node)

Struts BC and BD strength: $0.32f_{cu} = 0.32 \times 45 = 14.4$ MPa

Strut BC stress: $20885 \times 10^3/(1485 \times 1200) = 11.7$ MPa < 14.4 MPa (**OK**)

Strut BD stress: $13898 \times 10^3/(807 \times 1200) = 14.35$ MPa < 14.4 MPa (**OK**)

Node C (CCT node)

Nodal strength: $0.40mf_{cu} = 0.40 \times 1.414 \times 45 = 25.5$ MPa

Stress at the back face: $10591 \times 10^3/(613 \times 800) = 21.6$ MPa < 25.5 MPa (**OK**)

Strut BC strength: $0.32mf_{cu} = 0.32 \times 1.414 \times 45 = 20.4$ MPa

Strut BC stress: $20885 \times 10^3/(1345 \times 800) = 19.4$ MPa < 20.4 MPa (**OK**)

Node D (CTT node)

Nodal strength = Strut BD strength: $0.32f_{cu} = 0.32 \times 45 = 14.4$ MPa

Stress at the bearing face: $9000 \times 10^3/(1220 \times 1200) = 6.1$ MPa < 14.4 MPa (**OK**)

Strut BD stress: $13898 \times 10^3/(1257 \times 1200) = 9.2$ MPa < 14.4 MPa (**OK**)

8.6. Step 6: Determination of distributed reinforcement

The minimum required distributed reinforcement is 1200 x 1000 x 0.25% = 3000 mm^2/m. Adequate distributed reinforcement of T20-200 E.F. with an area of 3142 mm^2/m is provided in both ways. The reinforcement detail of the cantilever deep beam is shown in Figure 9.

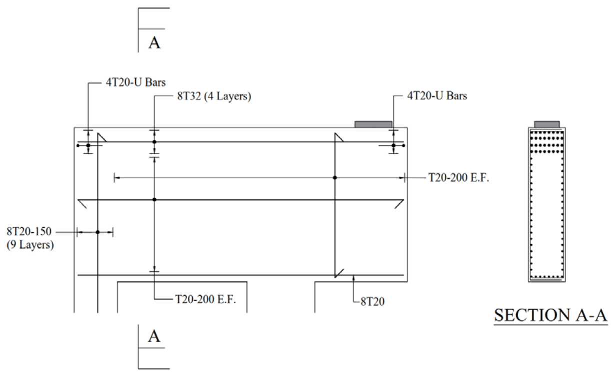

Figure 9. RC details of the cantilever deep beam designed using the STM.

9. Conclusion

In this paper, a review of the strength acceptance criteria for struts, nodes and bearings has been conducted, and the proposals in the literature and the provisions in selected design codes have been summarised and compared. Through calibration with the existing literature and design codes, unified STM design criteria with localisation have been proposed for application in Hong Kong. In the proposed design criteria, constant node efficiency factors for different nodal types, including CCC, CCT and CTT nodes, and constant normalised bearing strengths for different bearing conditions, namely dry bearing, bedded bearing and cast-in steel plate, are recommended. A constant strut efficiency factor is also proposed expressly for the non-hydrostatic nodal condition and the depth of compression zone is evaluated, based on the equilibrium of the STM system at shear failure and plastic stress state. As compared with the design criteria in selected design codes that may depend on multiple factors, such as strut angle and concrete strength, the proposed constant efficiency factors are more straightforward for practical use by engineers while maintaining a moderate conservatism. To align with the trend of the two recent design codes, AASHTO LRFD 2020 and ACI 318-19, it is recommended that the concrete confinement effect be included in the design criteria by adopting the confinement modification factor ($m = \sqrt{A_2/A_1} \le 2$). The STM design would become more economical by allowing more lenient design strength to be adopted when there is high confinement. Based on the proposed STM design criteria, a design example of a cantilever deep beam is prepared to demonstrate the STM design procedure.

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